Let $p=2rs$; $q=r^2+s^2$. Then $x=\dfrac{4r^2s^2}{r+s}$; $y=\dfrac{(r^2+s^2)^2}{r+s}$.

Let $r=k+l$; $s=k-l$. Then $x=\dfrac{2(k^2-l^2)^2}{k}$; $y=\dfrac{2(k^2+l^2)^2}{k}$.

Let $l=\alpha k$. Then $x=2k^3(1-\alpha^2)^2$; $y=2k^3(1+\alpha^2)^2$.

Now $a=p^3=8r^3s^3=8(k^2-l^2)^3=8k^6(1-\alpha^2)^3$, and $b=q^3=(r^2+s^2)^3=8(k^2+l^2)^3=8k^6(1+\alpha^2)^3$, where α and k are integers.

PROBLEMS.

53. Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.

Given $x^2-114\frac{1}{4}y^2=\mp 3$ to find the least values of x and y in integers.

54. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

In the expression $2x^2-2ax+b^2$, find two series of values for x in integral terms of a and b.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

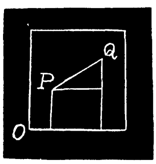
SOLUTIONS OF PROBLEMS.

35. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

Find the chance that the distance of two points within a square shall not exceed a side of the square. [From *Byerly's Integral Calculus.*]

I. Solution by ALWYN C. SMITH, The University of Colorado, Boulder, Colorado.

a is one side of the square ; P and Q the two points ; (x, y) the point P with O for origin ; and r and ϕ the polar coordinates of Q, with P as origin. Then the favorable cases are



$$4\int_0^{\frac{1}{4}\pi}\int_0^a\int_0^{a-r\sin\phi}\int_0^{a-r\cos\phi}dxdyrdrd\phi=a^4(\pi-\tfrac{13}{6}).$$

All the cases$=a^2.a^2=a^4$. Therefore, $p=\pi-\frac{13}{6}$.